IS THERE A SURJECTIVE RING HOMOMORPHISM
$R[[X]] \to R[X]$ **?**

All experts (I've asked) lean to a negative answer.
Explanations were given for this if R is local, or clean or potent.

Something similar is the following

Proposition 1. The zero ring is the only commutative ring R with $R[X] \cong R[[X]]$.

Proof: Assume $R[X] \cong R[[X]]$. It is known [Exercise **1.4** in [1], or **5.1** Theorem (Snaper) in [2]] that the Jacobson radical of $R[X]$ is equal to its nilradical. Thus, it would follow the same for $R[[X]]$. Since X lies in the Jacobson radical of $R[[X]]$ (in fact, $1 + Xf$ is invertible for all $f \in R[[X]]$), it follows that X is nilpotent, i.e. $X^n = 0$ for some $n \geq 0$. This shows $R = 0$.

If R is finite or countable, then $R[x]$ is countable, but $R[[x]]$ is uncountable.
1.4 In the ring $R[X]$ the Jacobson's radical equals the nilradical.
1.5 Let R be a ring and $R[[X]]$ the ring of formal power series $f = \sum_{n=0}^{\infty} a_n X^n$ with coefficients in R. Show that
(i) f is a unit in $R[[X]]$ iff $a_0 \in U(R)$;
(ii) If f is nilpotent then a_n is nilpotent for ever $n \geq 0$. Is the converse true ? [see below: Ch. 7, Ex. 2]
(iii) $f \in J(R[[X]])$ iff $a_0 \in J(R)$;
(iv) The contraction of a maximal ideal $\mathfrak{m}$ of $R[[X]]$ is a maximal ideal of R and $\mathfrak{m}$ is generate by $\mathfrak{m}^c$ and X;
(v) Every prime ideal or R is the contraction of a prime ideal of $R[[X]]$.

Here, if $\varphi : R \to S$ is a ring homomorphism, and $\mathfrak{b}$ is an ideal of S, then $\varphi^{-1}(\mathfrak{b})$ is called the *contraction* of $\mathfrak{b}$, denoted $\mathfrak{b}^c$.
Ch. 7, **Ex.2** Let R be a Noetherian ring and $f \in R[[X]]$. Then f is nilpotent iff each a_n is nilpotent.

However the answer is YES !
The solution was given by Yiqiang ZHOU.

Since $f : R[[X]] \to R$, given by $f(a + bX + cX^2 +,,,) = a$ is a retraction of rings (so more than surjective ring homomorphism), clearly it suffices to give an example of ring R such thar $R \cong R[X]$. Let S be any ring and let $R = S[X_1, X_2, ...]$. The $R[X] = S[X_1, X_2, ...][X] \cong S[X, X_1, X_2, ...] \cong S[X_1, X_2, ...] = R$, and we are done.

Notice that if $R \cong R[X_1]$, then $R[X_1] \cong R[X_1][X_2] = R[X_1, X_2]$, so $R \cong R[X_1, X_2]$. By induction $R \cong R[X_1, X_2, ..., X_n]$. This leads us to another
Question. Does it imply $R \cong R[X_1, X_2, ...]$?

References

[1] M. F. Atiyah, I. G. MacDonald *Introduction To Commutative Algebra* Addison-Wesley Series in Mathematics, 1969

[2] T. Y. Lam *A first course in noncommutative rings*. Sec ond Edition. Graduate texts in Math. 131, Springer New York Inc., 2001